STAT 2290 Exam 3A, April 2026, 9 problems, 75 minutes Name: Answers

**Read these instructions:**

- Leaving the testing room results in a new exam given for unfinished problems.
- Three detached sheets of notes allowed. **Turn in these notes with your exam.**
- No electronics.
- You may leave answers in terms of combinations, permutations, factorials, exponentiation, $\times$, $\div$, $+$, $-$, $\sqrt{\bullet}$ of numbers. For instance, $10 \times \sqrt{5 \times 271}/17$ is an acceptable answer.

**Useful critical values and truncated R outputs:**

| Confidence level $c$ | Critical value $z_c$ |
|---|---|
| 90% | 1.645 |
| 95% | 1.96 |
| 99% | 2.575 |

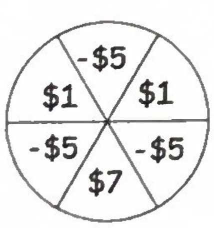
```
pnorm(-2) = 0.0227
pnorm(-1.5) = 0.0668
pnorm(-1) = 0.1586
pnorm(-0.5) = 0.3085
pnorm(0.5) = 0.6914
pnorm(1) = 0.8413
pnorm(1.5) = 0.9331
pnorm(2) = 0.9772
```

**Problem 1.** Estimate `pnorm(0, mean=3, sd=2)` from the above R outputs.

$$= \text{pnorm}\left(\frac{0-3}{2}\right) = \text{pnorm}(-1.5) = 0.0668$$

**Problem 2.** You throw a dart, hitting the wheel below. You win or lose the monetary amount indicated by where your dart lands. Let $X$ be the random variable modeling this monetary amount. Find the expected value and variance of $X$.

| $X = x$ | $-5$ | $1$ | $7$ |
|---|---|---|---|
| $P(X=x)$ | $\frac{3}{6}$ | $\frac{2}{6}$ | $\frac{1}{6}$ |




$$\Rightarrow E(X) = (-5)\cdot\frac{3}{6} + (1)\cdot\frac{2}{6} + (7)\cdot\frac{1}{6} = \frac{-15+2+7}{6} = -1$$

$$E(X^2) = (-5)^2\cdot\frac{3}{6} + (1)^2\cdot\frac{2}{6} + (7)^2\cdot\frac{1}{6} = \frac{75+2+49}{6} = \frac{126}{6} = 21$$

$$\Rightarrow \sigma_X^2 = E(X^2) - E(X) = (21) - (-1)^2.$$

**Problem 3.** Let $\hat{p}$ be the proportion of people in a random sample of 400 U.S. adults who say they drink the cereal milk. A spokesman for the dairy industry claims that 70% of all U.S. adults drink the cereal milk. Suppose this claim is true.

**(a)** What is the mean of the sampling distribution of $\hat{p}$ ?

$\mu_{\hat{p}} = p = 0.7$

**(b)** Find the standard deviation of the sampling distribution of $\hat{p}$ . (Justify your answer.)

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.7 \cdot 0.3}{400}}$ since 10% ~~Rate~~ Condition holds : there are ~~at least~~ $N \geq 10 \cdot n = 10 \cdot 400 = 4000$ US adults in the population

**(c)** Is the sampling distribution of $\hat{p}$ approximately normal? (Justify your answer.)

we check the Large Counts condition:

$n \cdot p = 400 \cdot 0.7 = 40 \times 7 = 280 \geq 10$ ✓

$n \cdot (1-p) = 400 \cdot 0.3 = 120 \geq 10$ ✓

So the sampling distribution of $\hat{p}$ is approx. normal.

**Problem 4.** The weights of a population of adult male penguins is known to be normally distributed with a population mean of 30 lbs. and a population standard deviation of 2 lbs.

**(a)** What is the probability that a randomly selected penguin from this population weighs less than 29 lbs.?

Here $n=1$, pop. is normal, so

$P(\text{weight} < 29 \text{ lbs}) = \text{pnorm}(29, \text{mean}=30, \text{sd}=2) = \text{pnorm}\left(\frac{29-30}{2}\right)$

$= \text{pnorm}(-0.5) = 0.3085 \quad (\approx 31\%)$

**(b)** What is the probability that the average weight of a random sample of 16 penguins from this population is less than 29 lbs.? (Justify your answer.)

Here $n=16$, pop normal so sampling dist of $\bar{x}$ is also normal.

There are $\geq 10 \cdot n = 10 \cdot 16$ adult male penguins so we can also compute $\sigma_{\bar{x}}$:

- $n=16$
- $\mu_{\bar{x}} = 30$
- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{16}} = \frac{1}{2}$

$\Rightarrow P(\bar{x} < 29) = \text{pnorm}(29, \text{mean}=30, \text{sd}=\frac{1}{2}) = \text{pnorm}\left(\frac{29-30}{1/2}\right) = \text{pnorm}(-2) = 0.0227$

or $\approx 2.3\%$.

**Problem 8.** The time to failure (in months) of a device is a random variable whose PDF is $f(x) = c/x^3$ if $x \geq 1$ and $f(x) = 0$ if $x < 1$.

**(a)** Find $c$.

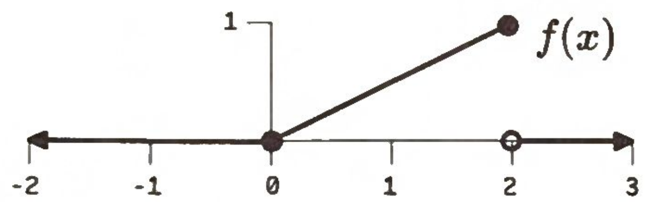
$cx^{-3}$

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_{1}^{\infty} \frac{c}{x^3}\,dx = \left[\frac{cx^{-2}}{-2}\right]_1^{\infty} = 0 - \frac{c}{-2} \Rightarrow \boxed{c = 2}$$

**(b)** Find the probability that the device lasts more than 2 months.

$$\int_{2}^{\infty} f(x)\,dx = \int_{2}^{\infty} \frac{2}{x^3}\,dx = \left[-x^{-2}\right]_2^{\infty} = 0 - \left[-\frac{1}{2^2}\right] = \frac{1}{4} = \boxed{25\% \text{ chance}}$$

**Problem 9.** A random variable $X$ has PDF $f(x)$ whose graph is shown:




**(a)** Find the CDF of $X$.

Know $f(x) = \begin{cases} 0 & x < 0 \\ x/2 \text{ if } & 0 \leq x \leq 2 \\ 0 & x > 2 \end{cases}$

If $0 \leq x \leq 2$, then $F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{0}^{x} \frac{x}{2}\,dx = \left.\frac{x^2}{4}\right]_0^x$

$= x^2/4 - 0.$

So $\boxed{F(x) = \begin{cases} 0 & x < 0 \\ x^2/4 \text{ if } & 0 \leq x \leq 2 \\ 1 & x > 2 \end{cases}}$

**(b)** Find the expected value and variance of $X$.

- $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{0}^{2} x \cdot \frac{x}{2}\,dx = \int_{0}^{2} \frac{x^2}{2}\,dx = \left[\frac{x^3}{6}\right]_0^2 = \frac{8}{6} = \boxed{\frac{4}{3}}$
- $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_{0}^{2} x^2 \cdot \frac{x}{2}\,dx = \int_{0}^{2} \frac{x^3}{2}\,dx = \left[\frac{x^4}{8}\right]_0^2 = 2.$

$\Rightarrow \sigma_X^2 = E(X^2) - E(X)^2 = \boxed{2 - \left(\frac{4}{3}\right)^2}$

| Page | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Points | 15 | 20 | 30 | 20 |
| Score | | | | |

**Problem 5.** A 2010 Gallup Poll asked a random sample of adults, "Are you satisfied or dissatisfied with the way things are going in the United States at this time?" Of the 100 respondents, 50 said that they were satisfied. Construct a 90% confidence interval for the proportion of adults who are satisfied with how things are going. (Justify your answer.) Large Counts : $n\hat{p} = 100 \cdot \frac{1}{2} = 50 \geq 10$, $n(1-\hat{p}) = 50 > 10$ ✓.

- Check 10% Condition: there are $\geq 10 \cdot 100 = 1000$ (adults) people in US. ✓

~~approx normal: $n = 100 \geq 30$ ✓~~

- $C = 90\%$ confidence $\Rightarrow z_c = 1.645$

$n = 100$

$\hat{p} = 50/100 = 0.5$

$E = \sqrt{\frac{0.5 \cdot 0.5}{100}} = \frac{0.5}{10} = 0.05$

So 90% confidence interval is $0.5 - 1.645 \times 0.05 \leq p \leq 0.5 + 1.645 \times 0.05$

- (optional) Interpretation: we are 90% confident that the true proportion of adults who are satisfied is between $0.05 - 1.645 \cdot 0.05$ & $0.5 + 1.645 \cdot 0.05$.

**Problem 6.** A manufacturer wants to estimate the mean radius of soccer balls within 0.1 cm. Determine the minimum sample size required to construct a 95% confidence interval for the population mean. Assume the population standard deviation is 0.5 cm. (Justify your answer.)

- $C = 95\% \Rightarrow z_c = 1.96$
- $\sigma = 0.5$,
- $0.1 \geq E = z_c \cdot \frac{\sigma}{\sqrt{n}} = \frac{1.96 \times 0.5}{\sqrt{n}} \xrightarrow{\text{Solve for } n} \sqrt{n} \geq \frac{1.96 \times 0.5}{0.1} = 1.96 \times 5$

$\Longrightarrow n \geq (1.96 \times 5)^2$

- So the min sample size needed is $\geq (1.96 \times 5)^2$ soccer balls.

**Problem 7.** Each of the 25 questions on an exam has multiple choice options A, B, C, D, and E. Let $X$ be the random variable that records the number of correct answers that result when you randomly guess on each of the 25 questions. $X$ is a binomial random variable with $n = 25$, $p = 1/5$, so ...

**(a)** Determine the expected value of $X$.

$E(X) = n \cdot p = 25 \cdot \frac{1}{5} = 5$

**(b)** Determine the standard deviation of $X$.

$\sigma_X = \sqrt{n(p)(1-p)} = \sqrt{25 \cdot \frac{1}{4} \cdot \frac{5}{4}}$